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Speed and shape of electron acoustic solitary waves in plasma

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Abstract : Large amplitude electron acoustic solitary waves are investigated in a two temperature plasma with finite ion-temperature. The Sagdeev's pseudopotential is derived in terms of cold electron speed (u_c). It is found that there exists a critical value of u_c (u_{cm}) at which $u_c' = 0$ beyond which the solitary waves cease to exist. This critical value also depends on T_c, the normalized cold electron temperature.

Keywords : Solitary wave, pseudopotential, electron inertia, soliton solution

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1. Introduction

Electron acoustic solitary waves (EAW) have been studied theoretically and experimentally by several authors [1-13] during the last three decades or so. In 1970, Arefev [1] and in 1973, Lashmore *et al* [3] first studied some important properties of EAW and showed that an electron acoustic wave can propagate quasi-perpendicular to *B*, the magnetic field in the magnetized plasma. Later, Watanabe and Taniuti [9], Yu and Shukla [10] also studied a modified EAW in an unmagnetized plasma. EAW are observed in unmagnetized plasma where the plasma consisted of two species of electrons with different temperatures called 'hot' and 'cold' electrons. EAW are becoming increasingly interesting because of their importance in astrophysical plasmas. Lin *et al* [11] have observed the two-electron population of widely disparate temperatures in the day side polar cusp. Baboolal *et al* [7] explained the broadband electrostatic noise in terms of the electron acoustic wave. Das *et al* [5] investigated the plasma with hot electron and hot and cold ions and they have shown that the rarefractive soliton solution exists in such a plasma, but no double layers solution exists. Guha *et al* [13] also studied the rarefractive electron acoustic solitary waves in two-electron plasma. Mace *et al* [8] investigated the same in a relativistic electron beam. The characteristics of the electrostatic turbulence generated by a gas of electron acoustic solitons were investigated by Dubouloz *et al* [2]. There are also other investigators who studied important properties and various applications of electron acoustic solitary waves [5,6,12]. These studies are restrictive in nature because most of them used reductive perturbative technique (RPT). Few years ago, Malfliet and Wieers [19] reviewed the studies on solitary waves in plasma and found that the RPT is based on the assumption of smallness of amplitude and so this technique can explain only small amplitude solitary waves. But large amplitude solitary waves also exist in nature. Nakamura *et al* [16] observed large amplitude solitary waves in laboratory plasma. So to study large amplitude solitary waves, one has to employ a non-perturbative approach. Sagdeev's [20] pseudopotential method is one such method to obtain exact solitary wave solution. This method has been successfully applied in various cases [21-23] including multi-component and multi-dimensional plasmas.

More recently, Johnston and Epstein [24] studied the nonlinear ion-acoustic solitary waves in a cold collisionless plasma by the direct analysis of the field equations. They observed that a very small (one millionth) change in the initial condition can destroy the oscillatory behaviour of the solitary

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waves. In this paper, our aim is to study large amplitude electron acoustic solitary waves in a plasma with warm ions and both hot and cold electrons. We will also study the role of the electron temperature on the existence of solitary waves. Recently, this technique was used by Maitra and Roychoudhury [25] and Chatterjee and Das ([26, 27]) to study the dust-acoustic solitary waves and ion acoustic solitary waves, respectively.

The organization of the paper is as follows. In Section 2, basic equations are written for one species of hot ion and both hot and cold electrons. The governing 2nd order ordinary differential equation is derived. Results and discussions are in Section 3 and Section 4 gives conclusions.

2. Basic equations

Our analysis is based on the continuity and momentum fluid equations for ions, electrons and Poisson's equation which are given below [8] :

$$\frac{\partial n_j}{\partial t}+\frac{\partial(n_j u_j)}{\partial x}=0, \tag{1}$$

$$\frac{\partial u_j}{\partial t}+u_j\frac{\partial u_j}{\partial x}+\frac{\mu_j}{n_j}\frac{\partial p_j}{\partial x}=-z_j\mu_j\frac{\partial\phi}{\partial x}, \tag{2}$$

$$\frac{\partial p_j}{\partial t}+u_j\frac{\partial p_j}{\partial x}+3p_j\frac{\partial u_j}{\partial x}=0, \tag{3}$$

$$\frac{\partial^2\phi}{\partial x^2}=n_{0h}e^{\phi}-\Sigma_j z_j n_j, \tag{4}$$

where $\mu_j=m_j/m_i$, $z_j=q_j/e$ and $j=i, c$ represent ion and cold electron, respectively. The subscript h denotes the hot electron. n_j's are the normalized ion and electron densities normalized to the total electron density n_{0e}, pressure to $n_{0e}T_h$. T_i and T_c are normalized ion and cold electron temperatures normalized to hot electron temperature T_h, electron potential ϕ to T_h/e, velocities to the hot electron thermal speed $\sqrt{T_h/m_e}$ [which is closely related to the ion sound speed $v_{se}=\sqrt{(n_{0c}/n_{0h})}v_h$], and masses to m_e. x and t are normalized to the Debye length $\lambda_D=\sqrt{\frac{kT_h}{4\pi n_{0e}e^2}}$ and ω_i^{-1}, respectively, where ω_i is the ion plasma frequency given by $\omega_i=\sqrt{\frac{4\pi n_{e0}e^2}{m_i}}$.

In order to search for solitary wave solutions for the eqs. (1) to (4), we introduce a linear substitution $\xi=x-vt$ assuming that solution to be a function of ξ. In the pseudopotential approach, several authors [21-23] used it. By substitution of $\frac{\partial u}{\partial x}=\frac{du}{d\xi}$ and $\frac{\partial u}{\partial t}=-v\frac{du}{d\xi}$, eqs. (1-4) reduce to,

$$-v\frac{dn_j}{d\xi}+\frac{d(n_j u_j)}{d\xi}=0, \tag{5}$$

$$-v\frac{du_j}{d\xi}+u_j\frac{du_j}{d\xi}+\frac{\mu_j}{n_j}\frac{dp_j}{d\xi}=-\frac{d\phi}{d\xi}, \tag{6}$$

$$-v\frac{dp_j}{d\xi}+u_j\frac{dp_j}{d\xi}+3p_j\frac{du}{d\xi}=0, \tag{7}$$

$$\frac{d^2\phi}{d\xi^2}=n_{0h}e^{\phi}+n_c-n_i. \tag{8}$$

To solve the above set of equations, the following boundary conditions are used : $\phi, \frac{d\phi}{d\xi}, u_i, u_c, u_h\to 0, n_i\to 1, n_c\to n_{0c}, p_i\to T_i, p_c\to n_{0c}T_c$ as $|\xi|\to\infty$. Integrating eq. (5) and using the boundary conditions given above, we get

$$n_c=\frac{vn_{0c}}{v-u_c}. \tag{9}$$

Now eliminating ϕ, n_c, n_i, p_c, p_i in terms of u_c, and keeping terms upto $O(\mu)$ we get

$$\frac{d^2u_c}{d\xi^2}=\frac{\partial\psi}{\partial u}, \tag{10}$$

where

$$\psi(u_c)=g(u_c)\left[\psi_h(u_c)+\psi_c(u_c)+\psi_i(u_c)\right], \tag{11}$$

when

$$g(u_c)=\frac{1}{\left[(v-u_c)\left(\frac{3T_c v^2}{(v-u_c)^4}-1\right)\right]^2}, \tag{12}$$

$$\psi_h(u_c)=n_{0h}(1-e^{v_1}), \tag{13}$$

$$\psi_c(u_c)=n_{0c}\left[vu_c+T_c\left(1-\frac{v^3}{(v-u_c)^3}\right)\right], \tag{14}$$

$$\psi_i(u_c)=v_1+\frac{\mu v_1^2}{2v^2}+\frac{3\mu T_i v_1}{4v^2}, \tag{15}$$

$$v_1=\left(-vu_c+\frac{u_c^2}{2}\right)\left(1-\frac{3T_c}{(v-u_c)^2}\right). \tag{16}$$

$\psi_h(u_c)$, $\psi_c(u_c)$ and $\psi_i(u_c)$ manifest the effects of n_h, n_c and n_i, respectively. Eq. (11) is derived using the technique employed in Ref [6]. All $\psi_h(u_c)$, $\psi_c(u_c)$ and $\psi_i(u_c)$ are obtained from eq (8) using eq. (6). Calculations are done neglecting terms of $O(\mu^2)$. Thus,

$$\frac{d^2u_c}{d\xi^2}=g_1(u_c)\left[n_{0h}e^{v_1}+\frac{n_{0c}v}{v-u_c}-1-\frac{\mu v_1}{v^2}-\frac{3\mu T_i}{4v^2}\right]$$

$$+g_2(u_e)\left[n_{0h}\left(1-e^{v_1}\right)+n_{0c}\left[vu_c+T_c\left(1-\frac{v^3}{(v-u_c)^3}\right)\right.\right.$$

$$\left.+v_1+\frac{\mu v_1^2}{2v^2}+\frac{3\mu T_i v_1}{4v^2}\right], \tag{17}$$

where

$$g_1(u_e)=\frac{1}{(v-u_e)^{-3}\left[3T_c v^2-(v-u_c)^4\right]}, \tag{18}$$

$$g_2(u_e)=2\frac{(v-u_e)^9+9T_c v^2(v-u_c)^5}{\left[3T_c v^2-(v-u_c)^4\right]^3}.$$

Also

$$\psi(u_c)=\frac{(u_c')^2}{2}. \tag{19}$$

3. Results and discussion

To find the region of existence of solitary waves, one has to study the nature of the function $\psi(u_e)$ and $\phi_1(u_c)$ defined by

$$\psi(u_c)=\frac{(u_c')^2}{2}, \tag{20}$$

where

$$u_c''=\frac{\partial\psi}{\partial u}$$
$$=\phi_1(u_c). \tag{21}$$

For solitary wave (see Refs. [25], [24]) $\phi_1(u_c)$ will have two roots, one being at $u_c=0$ and other at some point $u=u_{c1}(\geq 0)$. Also $\phi_1(u_c)$ should be positive on the interval $(0, u_{c1})$ and negative on (u_{c1}, u_{max}), where u_{max} is obtained from the non-zero root of $\psi(u_e)$. In Figure 1 $\phi_1(u_c)$ is plotted against u_c for different values of v. It is seen from the shape of the respective figures that the solitary waves would exist for $1 < v < 1.79$. Figure 2 shows the plot of $\psi(u_e)$ *vs* u_c for $v = 1.3$. Other parameters are $T_c = 0.01 = T_i$, $\mu = 1/1836$, $n_{0h} = 0.5$ and $n_{0c} = 0.5$. It is seen that $\psi(u_c)$ crosses the u_e-axis at $u_e = u_{0e} = 0.423062$ (up to 6 places of decimal). Hence, the amplitude of the solitary wave for this set of parameters will be 0.423062. To get the shape of the solitary wave, we have solved numerically the differential equation $u_c''=\psi_1(u_c)$ with $u_{0c}=0.423062$, $u_{0c}'=0$ and Figure 3(a) depicts the soliton solution $u_c(\xi)$ plotted against ξ. Other parameters are same as those in Figure 2. It is seen that

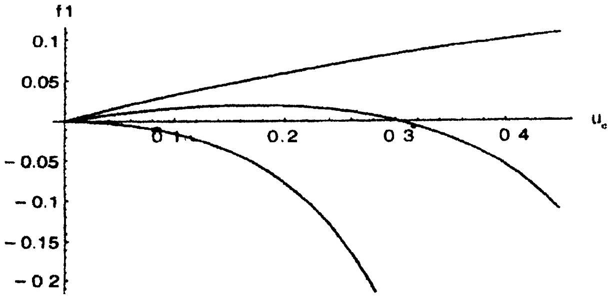

Figure 1. The plot of $\phi_1(u_e)$ *vs* u_c for $v = 1$, 1.3 and 1.79. Other parameters are $T_c = 0.01 = T_i$, $\mu = 1/1836$, $n_{0h} = 0.5$ and $n_{0c} = 0.5$.

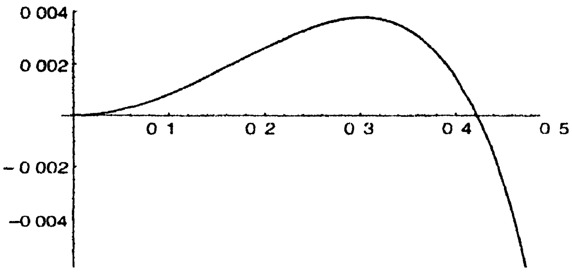

Figure 2. The plot of $\psi(u_c)$ *vs* u_c for $v = 1.3$. Other parameters are $T_c = 0.01 = T_i$, $\mu = 1/1836$, $n_{0h} = 0.5$ and $n_{0c} = 0.5$

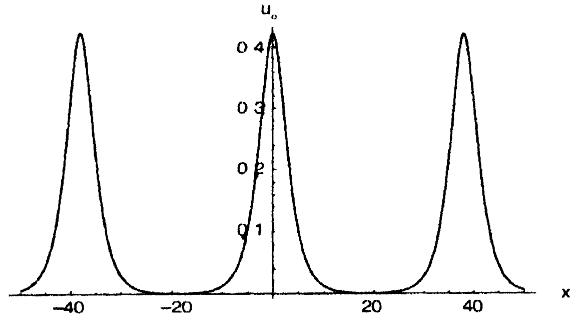

Figure 3(a). The soliton solution $u_c(\xi)$ plotted against ξ for $u_{0c} = 0.423062$. Other parameters are same as those in Figure 2

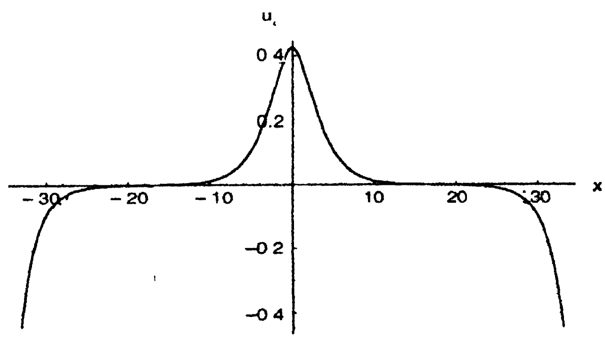

Figure 3(b). The soliton solution $u_c(\xi)$ plotted against ξ for $u_{0c} = 0.423063$. Other parameters are same as those in Figure 2.

$u_{0c} = 0.423062$ is the critical value for u_c. For $u > u_{0c}$, the soliton solution ceases to exist as can be seen in Figure 3(b). In this figure, u_{0c} is taken as 0.423063 (all other parameters are same as in Figure 3(a)). Hence, it is seen that for an extremely small change of value of u_{0c}, the periodic behaviour of the solitary wave is destroyed. Figure 4(a) depicts the soliton solution u_c plotted against ξ neglecting the effect of the electron mass.

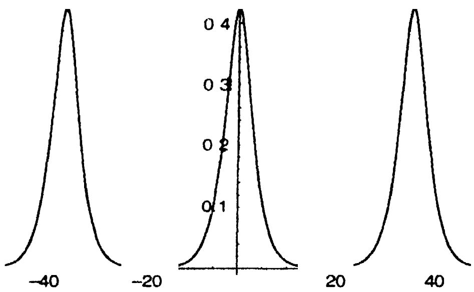

Figure 4(a). The soliton solution $u_c(\xi)$ plotted against ξ for u_{0c} = 0.423672 (neglecting electron inertia *i.e* $1/\mu \to \infty$) Other parameters are same as those in Figure 2

Other parameters are same as those in Figure 3(a). Here, u_{0c} is chosen as 0.423672 and this is the critical value for u_c for this case. In Figure 4(b), u_c is plotted against ξ and u_{0c} is chosen as 0.423673. Hence, here it is seen that the periodic behaviour of the soliton breaks at $u_c = 0.423673$ if one neglects the effect of the electron mass. But in Figures 3(a) and 3(b), it is seen that the

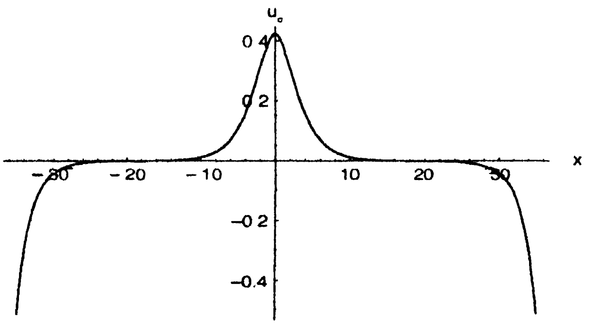

Figure 4(b). The soliton solution $u_c(\xi)$ plotted against ξ for u_{0c} = 0.423673. Other parameters are same as those in Figure 4(a).

periodic behaviour is destroyed at $u_{0c} = 0.423062$. Hence, the electron inertia has a non-insignificant effect on the periodic behaviour of the solitary waves. To see the effect of T_c on the critical value, Figure 5 is drawn. In Figure 5, $\psi(u_c)$ is plotted against u_c for different values of T_c, *viz*. $T_c = 0.01$ and 0.05. Other parameters are same as those in Figure 2. It is seen that the critical value of u_c decreases with the increase of T_c.

4. Conclusions

Using the pseudopotential approach, we have studied the speed and shape of the solitary waves. Sagdeev potential is obtained in terms of u_c, the cold electron velocity. It is seen that there exists a critical value of u_c at which $(u_c')^2 = 0$, beyond which the soliton solution would not exists. This critical value is

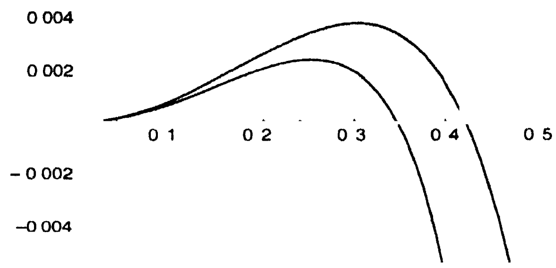

Figure 5. The plot of $\psi(u_c)$ vs u_c for T_c = 0.01 and 0 05. Other parameters are same as those in Figure 2. For the upper curve T_c = 0.01 and for the lower one T_c = .05

extremely sensitive to all parameters like, electron inertia or cold electron temperature. It is seen that the electron inertia and cold electron temperature plays significant roles in the forming and breaking of solitary waves.

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